Math 53: Multivariable Calculus **Sections 102, 103**

Worksheet for 2020-04-29

Problems

Problem 1. Throughout this problem, let H denote the plane $z = 2x + 4$.

(a) Let $\mathbf{F} = \langle 3yz, xz, xy - yz \rangle$. Show that if C is any oriented simple closed curve contained in the plane H, then $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$, regardless of C.

(b) Let $\mathbf{G} = \langle x^2 y - y, 0, y^3/6 \rangle$. If we let D to be any simple closed curve contained in the plane H which is oriented *counterclockwise* when viewed from above, find the maximum possible value of the integral $\int_D \mathbf{G} \cdot d\mathbf{r}$.

Problem 2. Use Stokes' theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle x^2 y, \frac{1}{3}x^3, xy \rangle$ and C is the curve of intersection of the hyperbolic paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$.

Problem 3. Consider the cone $z = \sqrt{x^2 + y^2}$, $z \leq 9$, oriented upwards. Use the divergence theorem to evaluate the flux of $\langle x, 0, 0 \rangle$ through the cone. Note that the cone is *not* a closed surface.

Problem 4. On a previous worksheet, we did the following problems by direct computation:

(a) Compute the flux of the vector field $\mathbf{F} = \langle x, y, z \rangle$ through the sphere $x^2 + y^2 + z^2 = 9$ oriented outwards.

(b) Compute the flux of the vector field $\langle 0, 0, 4 - z^2 \rangle$ outwards through the closed cylinder with lateral side $x^2 + y^2 = 10$ and lids $z = 0$ and $z = 2$.

Now do these problems again using the divergence theorem, and check that you get the same answers!